About the Ediz Eccentric connectivity index of Linear Polycene parallelogram Benzenoid

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ABSTRACT

A graph G=(V,E) consists of a set of vertices V(G) and a set of edges E(G). In chemical graphs, the vertices and edges of the graph correspond to the atoms of the molecule and chemical bonds, respectively. For a graph *G=(V, E)* , the *Eccentric Connectivity index* $\xi(G)$ is equal to $\xi(G)=\sum_{v\in V(G)} d_v\times\varepsilon(v)$ where $\varepsilon(v)$ is defined as the length of a maximal path connecting a vertex *v* to another vertex of *G*. In 2010, *S. Ediz et al.* defined *Ediz eccentric connectivity index* ${}^{E}\xi^{c}(G)$ of *G* and is defined as $\sum_{v\in V(G)} S(v)/\varepsilon(v)$, where *S(v)* is the sum of degrees of all vertices adjacent to vertex *v*. In this paper, we compute the Ediz eccentric connectivity index for an infinite family of linear Polycene parallelogram Benzenoid *P(n,n)* ($\forall n\geq 1$).

Keywords: Eccentric connectivity index, Ediz eccentric connectivity index, Molecular graphs, Linear Polycene parallelogram, Benzenoid.

1. INTRODUCTION

Let *G=(V,E)* be a simple connected graph, where *V(G)* and *E(G)* represents the set of vertices and set of edges, respectively. The distance, *d(u,v)*, is the length of a shortest path connecting the vertices *u* and *v*. The eccentricity of a vertex *v*, $\varepsilon(v)$, is the largest distance between *v* and any other vertex *u* of *G*. For a vertex *v*, $N_G(v)$ is the set of vertices adjacent to *v*. The degree of a vertex *v* denoted as d_v and $d_v=|N_G(v)|$.

Sharma, *Goswami* and *Madan* proposed the *eccentric connecitvity index* of a graph *G* [1]. It is defined as

$$\zeta^{c}(G)=\sum_{v\in V(G)} d_v\times\varepsilon(v)$$

It is often interesting to consider the sum of eccentricities of all vertices of a given graph G. This quantity called the *total eccentricity index* of the graph G.

$$\zeta(G)=\sum_{v\in V(G)}\varepsilon(v)$$

Gupta, *Singh* and *Madan* introduced the *connective eccentric index* of a graph G [2,3] as:

$$C^{\zeta}(G)=\sum_{v\in V(G)}\frac{d_v}{\varepsilon(v)}$$

In 2010, *Ediz* defined the *Ediz eccentric connectivity index* [4]defined as

$${}^{E}\zeta^{c}(G)=\sum_{v\in V(G)}\frac{S_v}{\varepsilon(v)}$$

Where S_v is the sum of degrees of all the vertices attached with v.
For further study and recent results about these indices see [4-12].

A general graph representation of linear polycence parallelogram benzenoid graph $P(m,n)$ has $2mn+2m+2n$ vertices and $3mn+2m+2n-1$ edges. A special case of this family is symmetric linear parallelogram benzenoid $P(n,n)$, shown in Figure 1. It has $2n(n+2)$ vertices and $3n^2+4n-1$ edges. For further study and more detail of this family of benzenoid reader can see references [13,14,15]

2. RESULTS AND DISCUSSION

The goal in this section is to compute the *Ediz Eccentric Connectivity index* for an infinite family of linear Polycene parallalogram of Benzenoid graph, as follows:

Theorem 1. [10] The Eccentric Connectivity index $\xi(G)$ of the linear Polycene parallalogram of Benzenoid $P(n,n)$ $(\forall n\in\mathbb{N}-\{1\})$ is equal to

$$\xi(P(n,n))=16n^3+15n^2-75n+6$$

Theorem 2. [10] The Connective Eccentric index $C^{\xi}(G)$ of the linear Polycene parallalogram of Benzenoid $P(n,n)$ $(\forall n\in\mathbb{N}-\{1\})$ is equal to

$$C^{\xi}(P(n,n))=\sum_{i=0}^{n-2}\left(\frac{5i^2+(16n+1)i+(11n+7)n}{2i^3+3(2n+1)i^2+(6n^2+6n+1)i+(2n^2+3n+1)n}\right)+\left(\frac{16n^2-10n+3}{n(8n^2+2n-1)}\right)$$

Theorem 3. Let $P(n,n)$ be the linear Polycene parallalogram of Benzenoid , thus the Ediz Eccentric Connectivity index ${}^{E}\xi^{c}(P(n,n))$ $(\forall n\in\mathbb{N}-\{1\})$ is

$${}^{E}\xi^{c}(P(n,n))=\sum_{i=1}^{n-2}\frac{12}{n+i}+\sum_{i=1}^{n-3}\frac{9(n-i)}{n+i}+\sum_{i=1}^{n-3}\frac{18(n-2-i)}{2n+2i+1}+\sum_{i=1}^{2n-3}\frac{28}{2n+i}+\left(\frac{9}{n-1}+\frac{6(3n-2)}{2n+1}+\frac{9(n-1)}{2n}+\frac{10}{2n-1}+\frac{8}{4n-1}\right)$$

Proof: Consider the linear Polycene Parallelogram of Benzenoid $P(n,n)$ with $2n(n+2)$ vertices and $3n^2+4n-1$ edges , where n $(\forall n\in\mathbb{N}-\{1\})$ be the number of hexagon C_6 in the first row/column of $P(n,n)$ (The general representation of linear Polycene parallalogram of

Benzenoid $P(n,n)$ is shown in Figure 1.). From the structure of $P(n,n)$ in Figure 1, we see that $4n+2$ vertices of $P(n,n)$ have degree 2 (we name a set of these vertices by V_2) and other $2n^2-2$ vertices have degree 3 (we name their set by V_3), and obviously

$$|V(P(n,n))|=|V_2|+|V_3|= [(4n+2)+(2n^2-2)] =2n(n+2)$$

$$|E(P(n,n))|= \frac{1}{2}[2\times|V_2|+3\times|V_3|]=\frac{1}{2}[2(4n+2)+3(2n^2-2)] = 3n^2+4n-1$$

From the structure of this Benzenoid graph $P(n,n)$ in Figure 1, we seee that the maximum and minimum eccentric of all vrtices in $V(P(n,n))$ are equal to [10 , 11]:

$$Max_{\varepsilon(v)}=4n-1$$
$$Min_{\varepsilon(v)}=2n.$$

In other words , by according to Figure 1 and results from refences [10,11,12] , one can see that (See Figure 1 and Tabel 1.):

$$\forall v\in V(P(n,n));\ \ 2n\leq\varepsilon(v)\leq 4n-1$$
$$\forall v\in V_2\subset V(P(n,n));\ \ \varepsilon(v)\in\{4n-1, 4n-2, 4n-4, 4n-6,\ldots, 2n+2, 2n+1\}$$
$$\forall v\in V_3\subset V(P(n,n));\ \ \ 2n\leq\varepsilon(v)\leq 4n-3$$

Tabel 1. [10-12] All Eccentric of vertices in the linear Polycene parallalogram Benzenoid graph $P(n,n)$.

$2n+1$	$2n+1$	$2n+2$	…	$4n-5$	$4n-4$	$4n-3$	$4n-2$	$4n-1$
$2n$	$2n+1$	$2n+2$	…	$4n-5$	$4n-4$	$4n-3$	$4n-2$	
$2n$	$2n+1$	$2n+2$	…	$4n-5$	$4n-4$			
$2n$	$2n+1$	$2n+2$	…					
…	…	…						
…	…	…						
…	…	…						
$2n$	$2n+1$	$2n+2$						
	$2n+1$							

Now, to compute the summation of degrees of all neighbors of vertices in in the linear Polycene parallalogram Benzenoid graph $P(n,n)$, we see that 4 of vertecs as degree 2 (in $V_2\subset V(P(n,n))$ are adjacent with a member of V_2 and a member of V_3 , thus the summation of degrees of these 4 vertiucse is 2+3=5 where the eccentric of these vertices is $2n+1$. We know that $\forall u\in V(P(n,n));\ N_G(u)=\{v\in V(G)|uv \text{ in } E(G)\}$ and $S(u)=\sum_{v\in N_G(u)} d_v$.

Also, only 2 vertece of V_2 are adjacent with two members of V_2 and $S(v)$ =4 (The eccentric of these vertices is $4n-1$). Form Figure 1, we see that all other members ($4n-4$ vertices) of V_2 are adjacent with members of V_3 , the summation of degrees is equal to 6 and for the eccentric of them , we have $2n+2\leq\varepsilon(v)\leq 4n-2$.

For vertece of V_3; we see that $4(n-1)$ outer vertece (with degree 3) are adjacent with with two members of V_2 and an inner member of V_3 , therefore , the summation of degrees will be 2+2+3=7. Finally , for $2n^2-2-4(n-1)$ inner vertece of V_3, the summation of degrees is equal to 3×3=9.

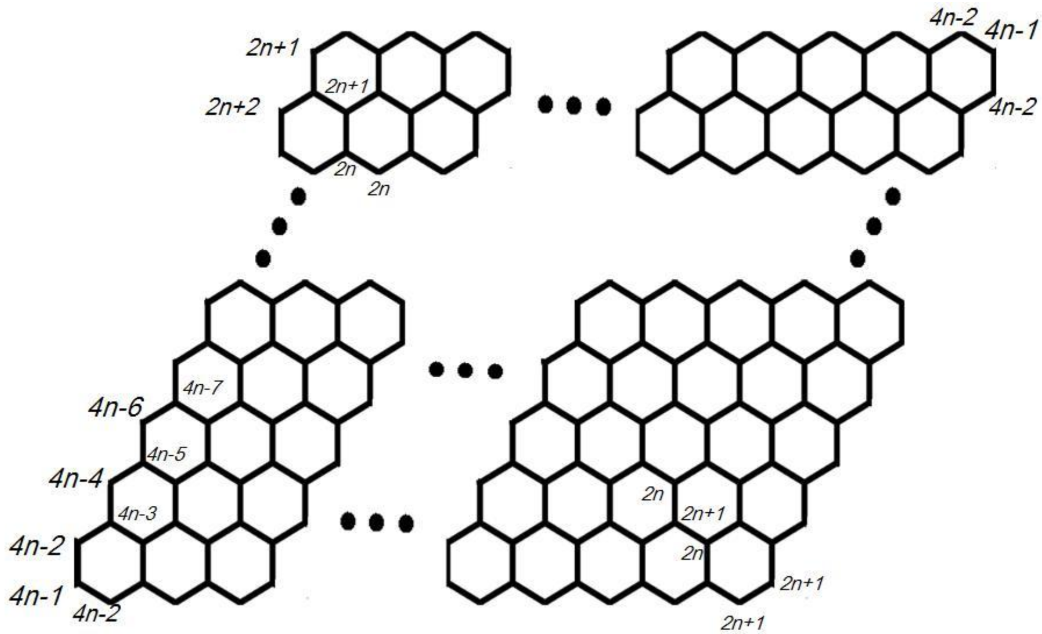

Figure 1. [10-12] The eccentric of vertices of linear Polycene parallelogram of Benzenoid *P(n,n)*.

Now, we can compute the Ediz eccentric connectivity index of an infinite family of linear Polycene parallelogram of Benzenoid *P(n,n)* ($\forall n\geq 1$) as follow:

$$
\begin{aligned}
{}^{E}\zeta^{c}(P(n,n)) &= \sum_{v\in V(P(n,n))}\frac{S(v)}{\varepsilon(v)}\\
&= 2\left(\frac{4}{4n-1}\right)+4\left(\frac{5}{4n-2}\right)+4\left(\frac{6}{4n-4}\right)+4\left(\frac{6}{4n-6}\right)+\ldots+4\left(\frac{6}{2n+4}\right)+4\left(\frac{6}{2n+2}\right)+4\left(\frac{6}{2n+1}\right)\\
&+2(n-1)\left(\frac{9}{2n}\right)+2(n-2)\left(\frac{9}{2n+1}\right)+4\left(\frac{7}{2n+1}\right)+2(n-1)\left(\frac{9}{2n+2}\right)+2(n-3)\left(\frac{9}{2n+3}\right)+4\left(\frac{7}{2n+3}\right)\\
&+2(n-2)\left(\frac{9}{2n+4}\right)+2(n-4)\left(\frac{9}{2n+5}\right)+4\left(\frac{7}{2n+5}\right)+\ldots+8\left(\frac{9}{4n-8}\right)+4\left(\frac{9}{4n-7}\right)+4\left(\frac{7}{4n-7}\right)\\
&+6\left(\frac{9}{4n-6}\right)+2\left(\frac{9}{4n-5}\right)+4\left(\frac{7}{4n-5}\right)+4\left(\frac{9}{4n-4}\right)+4\left(\frac{7}{4n-3}\right)\\
&= \frac{8}{4n-1}+\frac{10}{2n-1}+\frac{12}{2n-2}+\frac{12}{2n-3}+\ldots+\frac{12}{n+2}+\frac{12}{n+1}+\frac{24}{2n+1}\\
&+\frac{9(n-1)}{2n}+\frac{18(n-2)}{2n+1}+\frac{9}{n-1}\\
&+\frac{18(n-1)}{2n+2}+\frac{18(n-3)}{2n+3}+\frac{18(n-2)}{2n+4}+\frac{18(n-4)}{2n+5}+\ldots+\frac{72}{4n-8}+\frac{36}{4n-7}+\frac{54}{4n-6}+\frac{18}{4n-5}\\
&+28\left(\frac{1}{2n+1}+\frac{1}{2n+3}+\frac{1}{2n+5}+\ldots+\frac{1}{4n-7}+\frac{1}{4n-5}+\frac{1}{4n-3}\right)\\
&= \frac{8}{4n-1}+\frac{24}{2n+1}+\frac{10}{2n-1}+12\sum_{i=1}^{n-2}\frac{1}{n+i}
\end{aligned}
$$

$$+\frac{9(n-1)}{2n}+\frac{18(n-2)}{2n+1}+\frac{9}{n-1}+18\sum_{i=1}^{n-3}\frac{n-i}{2n+2i}+18\sum_{i=1}^{n-3}\frac{n-2-i}{2n+2i+1}+28\sum_{i=1}^{2n-3}\frac{1}{2n+i}$$

Here the Ediz eccentric connectivity index of linear Polycene parallelogram of Benzenoid *P(n,n)* is equal to

$${}^{E}\xi^{c}\left(P(n,n)\right)=\sum_{i=1}^{n-2}\frac{12}{n+i}+\sum_{i=1}^{n-3}\frac{9(n-i)}{n+i}+\sum_{i=1}^{n-3}\frac{18(n-2-i)}{2n+2i+1}+\sum_{i=1}^{2n-3}\frac{28}{2n+i}+\left(\frac{9}{n-1}+\frac{6(3n-2)}{2n+1}+\frac{9(n-1)}{2n}+\frac{10}{2n-1}+\frac{8}{4n-1}\right)$$

Now the proof of Theorem 3 was completed. ■

Example 1. Consider the graph of linear Polycene parallelogram of Benzenoid *P(2,2)* depicted in Figure 2. This graph has 16 vertices and 19 edges. And the Ediz eccentric connectivity index of *P(2,2)* is ${}^{E}\xi^{c}\left(P(2,2)\right)=\frac{28}{5}+\left(\frac{9}{1}+\frac{24}{5}+\frac{9}{4}+\frac{10}{3}+\frac{8}{7}\right)=26.126$.

Example 2. Consider the linear Polycene parallelogram of Benzenoid *P(3,3)* depicted in Figure 2. By Theore 3, its Ediz eccentric connectivity index is equal to

$${}^{E}\xi^{c}\left(P(3,3)\right)=\frac{12}{4}+\sum_{i=1}^{3}\frac{28}{6+i}+\left(\frac{9}{2}+\frac{6(7)}{7}+\frac{9(2)}{6}+\frac{10}{5}+\frac{8}{11}\right)=29.8\overline{3}.$$

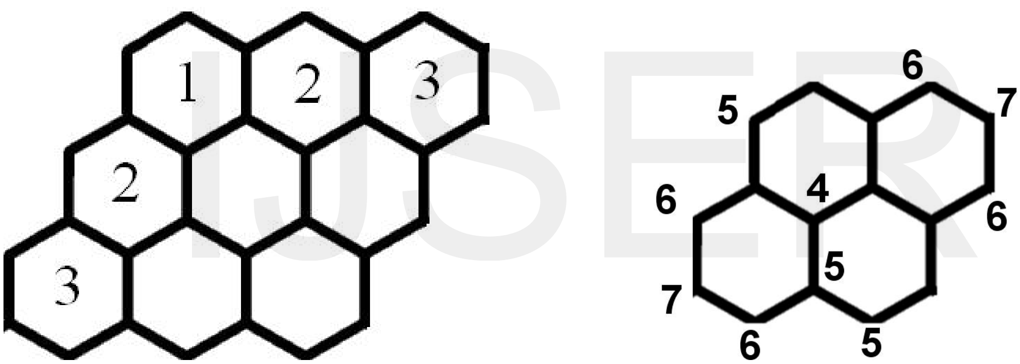

Figure 1. Examples of two first members of the linear Polycene parallelogram of Benzenoid.

Example 3. Let *P(n,n)* be the linear Polycene parallelogram of Benzenoid *(∀n∈ℕ-{1})* depicted in Figure 1. In following table (Table 2), we presnt some values of its Ediz eccentric connectivity index ${}^{E}\xi^{c}(P(n,n))$ *for* integers *n*=2, 3,..., 1000000.

Table 1: Some values of the Ediz eccentric connectivity index ${}^{E}\xi^{c}(P(n,n))$ for integer *k=2, 3,..., 20000000*.

integer *n*	the Ediz eccentric connectivity index ${}^{E}\xi^{c}(P(n,n))$
2	26.1261904761905
3	29.8383838383838
4	39.3836802086802
5	47.8100916970886
6	55.7128830801869
7	63.3288354255537
8	70.7707465326409
9	78.0991614201268
10	85.3494772260268
11	92.54376625314
12	99.6965040272753
13	106.817577336281
14	113.913968298426
15	120.990747701208
16	128.051686910272
17	135.099648907114

18	142.136846169013
19	149.16501543959
20	156.185539039241
30	226.14079967336
40	295.882670281653
50	365.54026712877
60	435.156027119135
70	504.747990443711
80	574.32512899922
90	643.892408143483
100	713.45279853161
200	1408.90564365205
300	2104.27637237402
400	2799.62663931005
500	3494.96873229991
600	4190.30674130764
700	4885.64241770999
800	5580.97663671463
900	6276.30988435815
1000	6971.64245217707
2000	13924.9531791595
3000	20878.2557537294
4000	27831.5562908691
5000	34784.8560131445
6000	41738.1553280174
7000	48691.4544100997
8000	55644.7533466934
9000	62598.0521862964
10000	69551.3509580077

20000	139084.337181543
30000	208617.322590431
40000	278150.307795665
50000	347683.292919437
60000	417216.27800247
70000	486749.26306224
80000	556282.248107461
90000	625815.233142977
100000	695348.218171716
200000	1390678.06830966
300000	2086007.91836623
400000	2781337.76840241
500000	3476667.61843043
600000	4171997.46845447
700000	4867327.31847603
800000	5562657.16849636
900000	6257987.0185154
1000000	6953316.86853387
2000000	13906615.3687043
3000000	20859913.8688663
4000000	27813212.3690284
5000000	34766510.8691845
6000000	41719809.3693448
7000000	48673107.8695061
8000000	55626406.3696612
9000000	62579704.8698186
10000000	69533003.3699899
20000000	139065988.371589

Corollary 1. Consider the graph of linear Polycene parallelogram of Benzenoid $P(n,n)$ depicted in Figure 2. This graph has $2n(n+2)$ vertices and $3n^2+4n-1$ edges $(\forall n \in \mathbb{N}-\{1\})$. By using Theorem 1 and results from Table 2, we conclude for enough large integer number n,k, we have following approach for the Ediz eccentric connectivity index ${}^{E}\xi^{c}(P(n,n))$:

(1). $\forall n = 10^k;\ {}^{E}\xi^{c}(P(n,n)) \simeq 6.95\times10^{k}$
(2). $\forall n = 2\times10^k;\ {}^{E}\xi^{c}(P(n,n)) \simeq 1.39\times10^{k+1}$
(3). $\forall n = 3\times10^k;\ {}^{E}\xi^{c}(P(n,n)) \simeq 2.08\times10^{k+1}$
(4). $\forall n = 4\times10^k;\ {}^{E}\xi^{c}(P(n,n)) \simeq 2.78\times10^{k+1}$
(5). $\forall n = 5\times10^k;\ {}^{E}\xi^{c}(P(n,n)) \simeq 3.47\times10^{k+1}$
(6). $\forall n = 6\times10^i;\ {}^{E}\xi^{c}(P(n,n)) \simeq 4.17\times10^{k+1}$
(7). $\forall n = 7\times10^k;\ {}^{E}\xi^{c}(P(n,n)) \simeq 4.86\times10^{k+1}$
(8). $\forall n = 8\times10^k;\ {}^{E}\xi^{c}(P(n,n)) \simeq 5.56\times10^{k+1}$
(9). $\forall n = 9\times10^k;\ {}^{E}\xi^{c}(P(n,n)) \simeq 6.25\times10^{k+1}$

Corollary 2. By according to above corollary and Table 2, we can imply that

(1). $^{E}\xi^{c}(P(3\times10^{k}, 3\times10^{k})) \simeq 1.49\times {}^{E}\xi^{c}(P(2\times10^{k}, 2\times10^{k}))$

(2). $^{E}\xi^{c}(P(4\times10^{k}, 4\times10^{k})) \simeq 1.33\times {}^{E}\xi^{c}(P(3\times10^{k}, 3\times10^{k}))$

(3). $^{E}\xi^{c}(P(5\times10^{k}, 5\times10^{k})) \simeq 1.24\times {}^{E}\xi^{c}(P(4\times10^{k}, 4\times10^{k}))$

(4). $^{E}\xi^{c}(P(6\times10^{k}, 6\times10^{k})) \simeq 1.20\times {}^{E}\xi^{c}(P(5\times10^{k}, 5\times10^{k}))$

(5). $^{E}\xi^{c}(P(7\times10^{k}, 7\times10^{k})) \simeq 1.16\times {}^{E}\xi^{c}(P(6\times10^{k}, 6\times10^{k}))$

(6). $^{E}\xi^{c}(P(8\times10^{k}, 8\times10^{k})) \simeq 1.14\times {}^{E}\xi^{c}(P(7\times10^{k}, 7\times10^{k}))$

(7). $^{E}\xi^{c}(P(9\times10^{k}, 9\times10^{k})) \simeq 1.12\times {}^{E}\xi^{c}(P(8\times10^{k}, 8\times10^{k}))$

(8). $^{E}\xi^{c}(P(10^{k+1}, 10^{k+1})) \simeq 1.11\times {}^{E}\xi^{c}(P(9\times10^{k}, 9\times10^{k}))$.

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